By angular momentum conservation about the observer placed at the 'hole' point,

$$r^2\omega = \text{const.}$$

$$2r(dr)\omega + r^2 d\omega = 0$$

$$d\omega = -2\left(\frac{dr}{r}\right)\omega$$. Initially, tension will be $\underline{\underline{m\omega^2 r = Mg}}$.

Now, suppose 'M' mass is pulled slightly downwards. Let's choose an observer at 'hole', rotating with

same omega as omega of the string. Then, FBD of particle wrt rotating observer will be



$$m\left[\omega - 2\frac{dr}{r}\omega\right]^2 (r-dr) - T = ma.$$

$$m\omega^2 r\left(1-\frac{dr}{r}\right)^3 - T = ma$$

$$m\omega^2 r\left(1-3\frac{dr}{r}\right) - T = ma$$

&

$$T - Mg = Ma$$

Adding up,

$$m\omega^2 r - 3m\omega^2 dr - Mg = (M+m)a$$

But $Mg = m\omega^2 r$ as expressed in equilibrium condition. Hence,

$$a = -\left(\frac{3m\omega^2}{(M+m)}\right) dr.$$

$$T = 2\pi \sqrt{\frac{(M+m)}{3m\omega^2}}$$